Two or more independent variables (factors)

Factorial ANOVA

Data files

- Gueguen(2012).sav
- Classdata.sav

Homework: Davey(2003).sav

Definitions

- **Factorial ANOVA**
  - An ANOVA with more than one discrete IV and one scale DV. Each IV is called a factor.

- **Main Effect**
  - The omnibus test of a single factor ignoring any other factors is called a main effect.

- **Interaction**
  - Test of the interaction between the two IVs
  - A relationship involving three variables in which the relationship between two of the variables differs depending upon the third variable. An interaction effect exists when the relationship between two variables changes for different values of a third variable.

Main effect

- Main effects
  - Overall effect of an IV in a factorial design
    - effect on DV *as if* only that IV was studied
- The main effect of a factor is the effect that changing the levels of that factor has on dependent variable scores while ignoring all other factors in the study
- We collapse across a factor. Collapsing across a factor means averaging together all scores from all levels of that factor.

Main Effect: An outcome that represents a consistent difference between levels of a factor

SS_{M1} = difference between the means of each level of Factor1 and the grand mean

SS_{M2} = difference between the means of each level of Factor2 and the grand mean

Interaction

- Interaction effects
  - Combined effect of IVs considered simultaneously
  - An interaction effect occurs when the effect of an independent variable differs depending on the level of a 2nd independent variable.
- The interaction of two factors is called a two-way interaction
  - The two-way interaction effect is the influence on scores that results from combining the levels of factor A with the levels of factor B
  - When you look for the interaction effect, you compare the cell means. When you look for a main effect, you compare the level means.

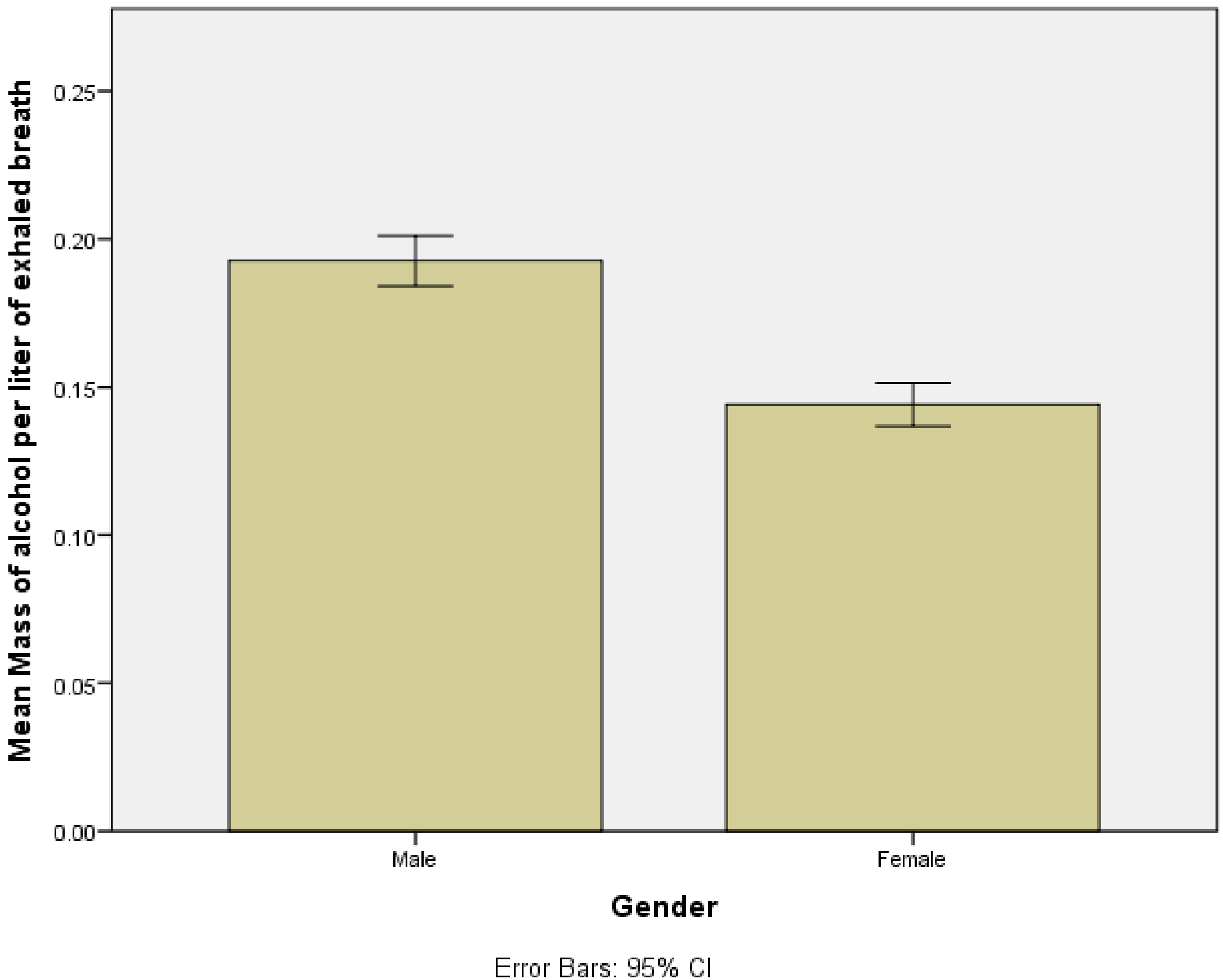
Example: Gueguen(2012).sav

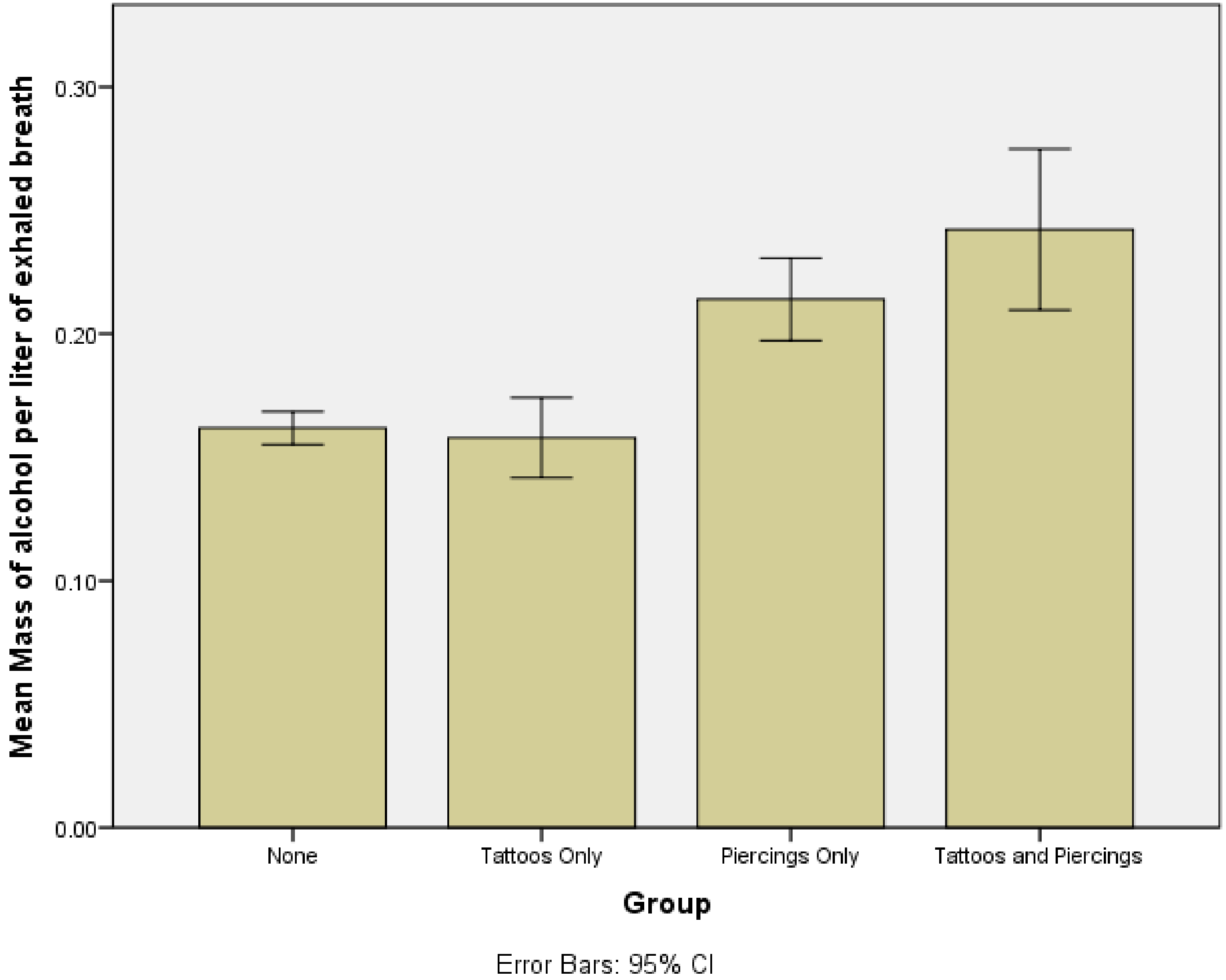
Is risk taking behaviour related to tattoos and body piercings? Does this relationship depend on gender?

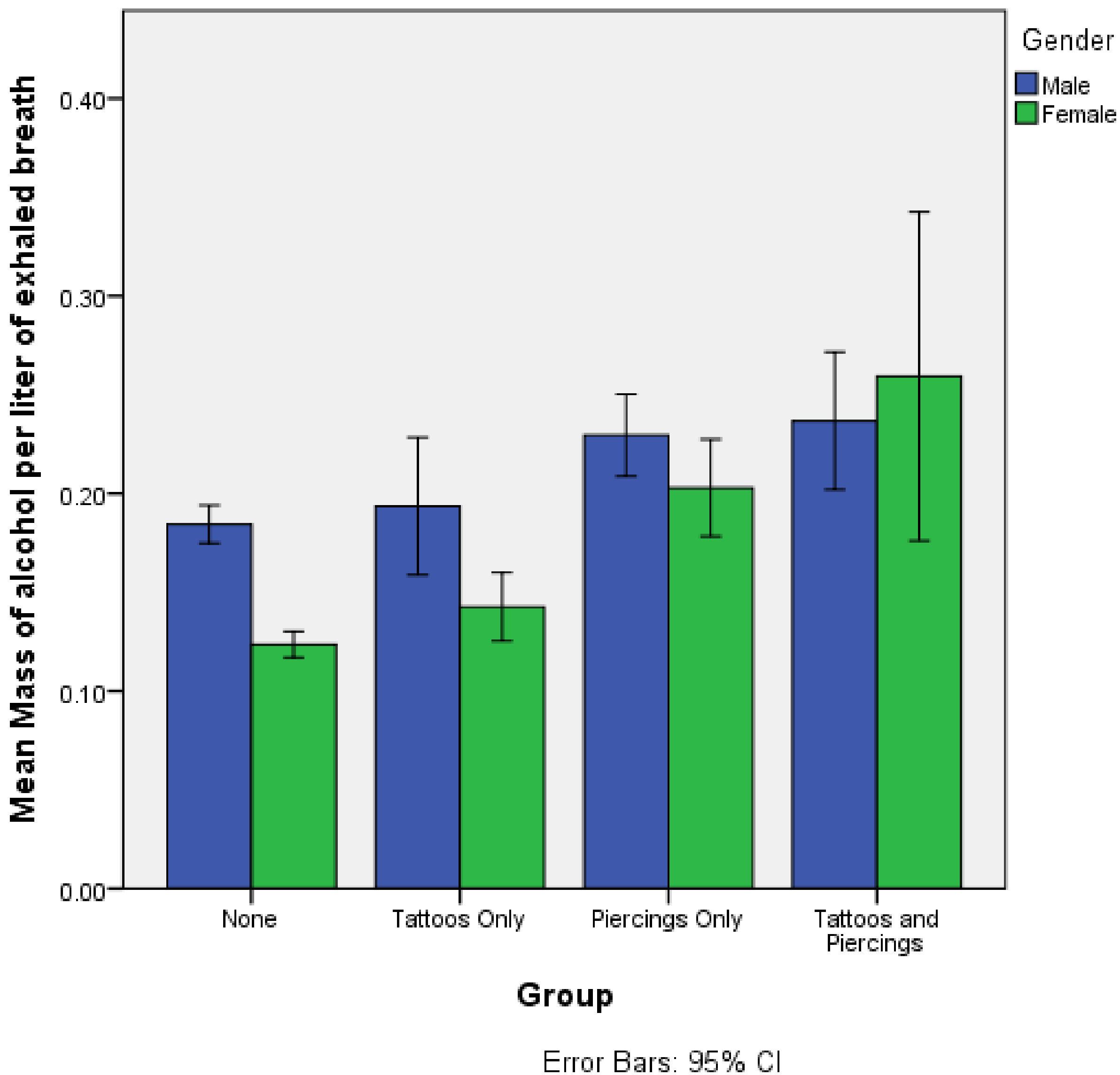
- gender
  - Men and women
- Piercings
  - None, tattoos, piercings, tattoos and piercings
- Blood alcohol level measured when coming out of a bar

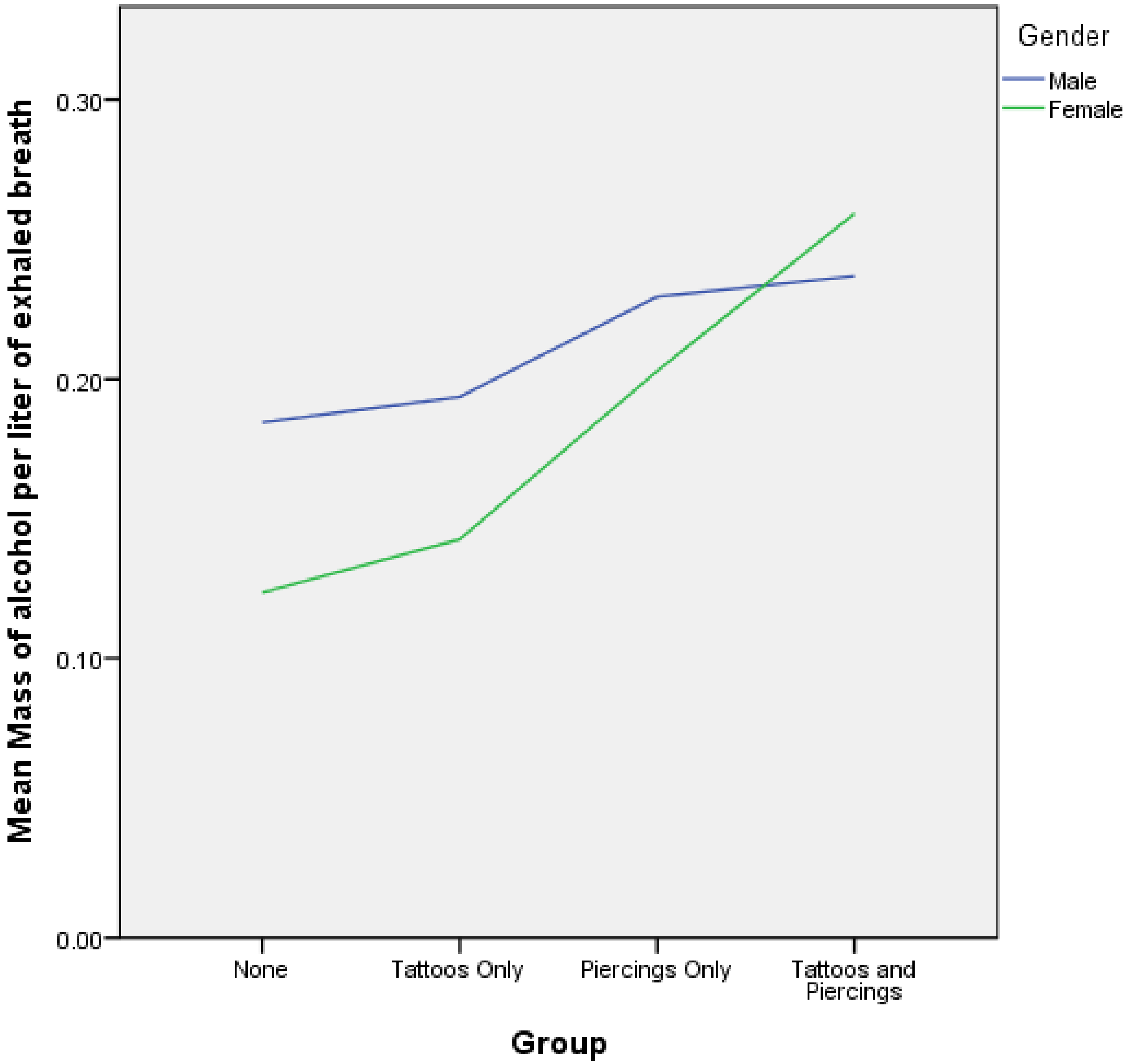
Example

- Factorial combination of 2 IVs
  - Gender
  - Body decoration
  - Factorial combination: 8 conditions
  - Referred to as a 2 x 4 (first IV with two levels, second IV with four levels)
- What we can look at:
  - Effect of Factor A (main effect of Gender)
  - Effect of Factor B (main effect of Body Decoration)
  - Interaction
    - Does the effect of one factor depend on the level of the other factor? Does the effect of body decorations depend on gender?


Mean Mass of alcohol per liter of exhaled breath
0.25
0.20
0.15
0.10
0.05
0.00
Male
Female
Gender
Error Bars: 95% CI


Mean Mass of alcohol per liter of exhaled breath
0.30
0.20
0.10
0.00
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group
Error Bars: 95% CI

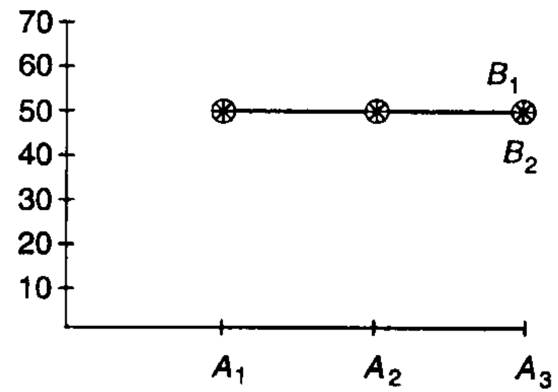

Mean Mass of alcohol per liter of exhaled breath
0.00
0.10
0.20
0.30
0.40
Gender
Male
Female
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group
Error Bars: 95% CI



Gender
Male
Female
0.30
0.20
0.10
0.00
Mean Mass of alcohol per liter of exhaled breath
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group

Possible Outcomes

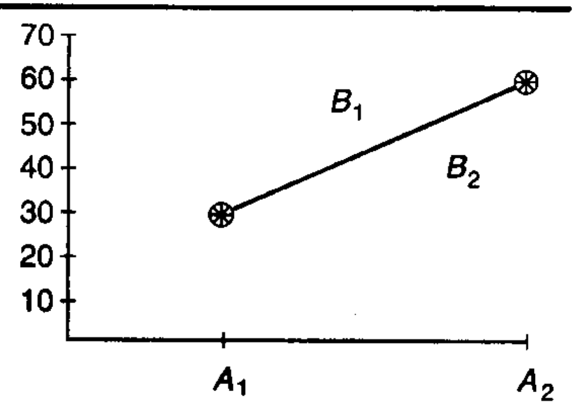
	A_1	A_2	A_3	Mean
B_1	50	50	50	50
B_2	50	50	50	50
Mean	50	50	50	



No effects

(a) 3 × 2 Factorial (*A*, *B*, and the interaction are not significant)

	A_1	A_2	Mean
B_1	30	60	45
B_2	30	60	45
Mean	30	60	

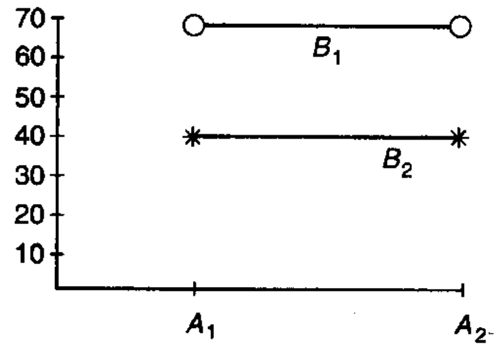


Main effect of A

(b) 2 × 2 Factorial (*A* is significant; *B* and the interaction are not significant)

Possible Outcomes

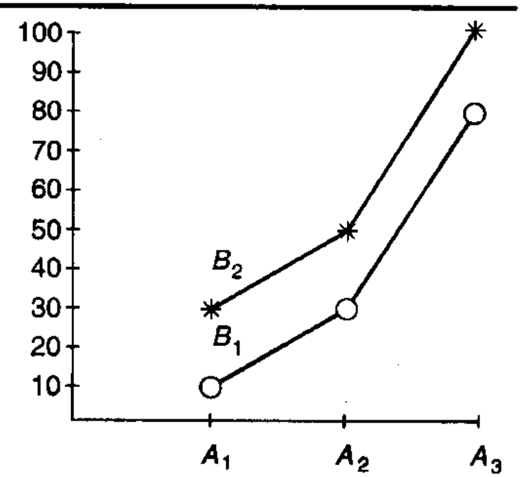
	A_1	A_2	Mean
B_1	70	70	70
B_2	40	40	40
Mean	55	55	



(c) 2 × 2 Factorial (*B* is significant; *A* and the interaction are not significant)

Main effect of B

	A_1	A_2	A_3	Mean
B_1	10	30	80	40
B_2	30	50	100	60
Mean	20	40	90	

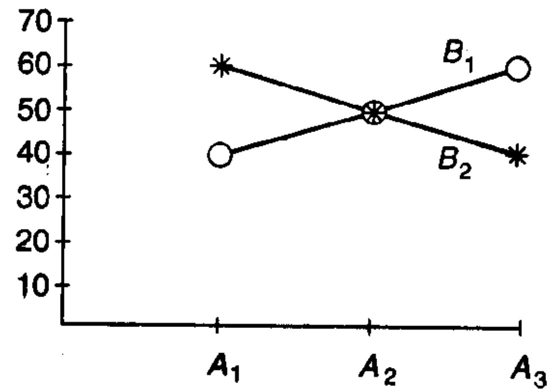


(d) 3 × 2 Factorial (*A* and *B* are significant; the interaction is not significant)

Main effect of A and B

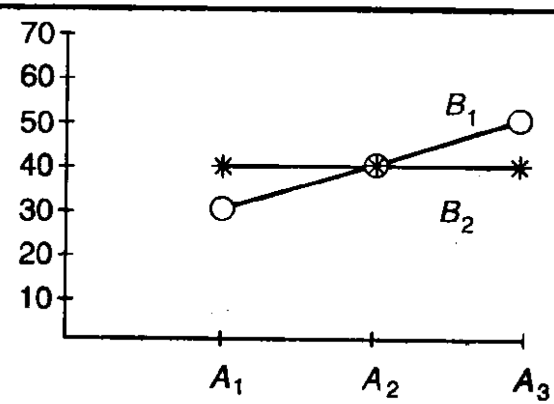
Possible Outcomes

	A_1	A_2	A_3	Mean
B_1	40	50	60	50
B_2	60	50	40	50
Mean	50	50	50	



(e) 3 × 2 Factorial (the interaction is significant: *A* and *B* are not significant)

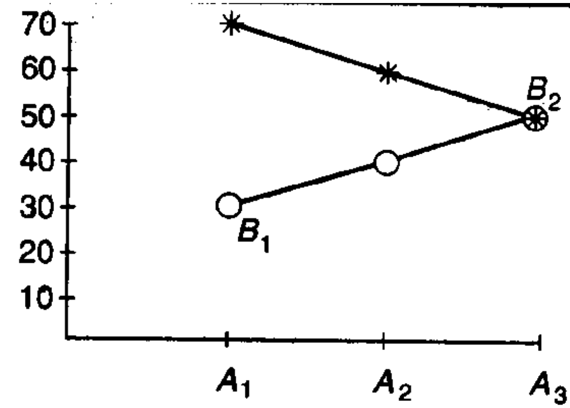
	A_1	A_2	A_3	Mean
B_1	30	40	50	40
B_2	40	40	40	40
Mean	35	40	45	



(f) 3 × 2 Factorial (*A* and the interaction are significant; *B* is not significant)

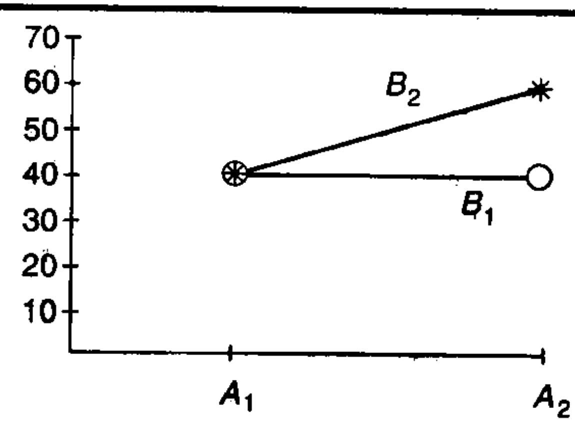
Possible Outcomes

	A_1	A_2	A_3	Mean
B_1	30	40	50	40
B_2	70	60	50	60
Mean	50	50	50	



(g) 3 × 2 Factorial (*B* and the interaction are significant; *A* is not significant)

	A_1	A_2	Mean
B_1	40	40	40
B_2	40	60	50
Mean	40	50	



(h) 2 × 2 Factorial (*A*, *B*, and the interaction are significant)

Factorial ANOVA

- ANOVA can analyze any factorial design
- The number of effects will depend on the number of independent variables (IVs)
  - 2 IVs: A & B main effects; AB interaction
  - 3 IVs: A, B, & C main effects; AB, AC, BC, & ABC interactions
  - 4 IVs: A, B, C, & D main effects; AB, AC, AD, BC, BD, CD, ABC, ABD, BCD, & ABCD interactions
  - Etc.

FORMALLY

Assumptions

The two-way independent groups ANOVA test requires the following statistical assumptions:

1. Random and independent sampling.

2. Data are from normally distributed populations.
   Note: This test is robust against violation of this assumption if $n \geq 30$ for all groups.

3. Variances in these populations are roughly equal (Homogeneity of variance).
   Note: This test is robust against violation of this assumption if all group sizes are equal.

The maths behind ANOVA: A Quick Review

Analysis of variance (ANOVA) is a method of testing the null hypothesis that three or more means are roughly equal.

ANOVA produces a test statistic termed the *F-ratio*.

$$\frac{\text{Systematic Variance } (SS_M)}{\text{Unsystematic Variance } (SS_R)}$$

The F-ratio tells us only that the experimental manipulation has had an effect—not where the effect has occurred.

-- Planned comparisons

-- Post-hoc tests

-- Purpose of follow-up tests? Control Type I error rate at 5%.

Separating Out The Variance

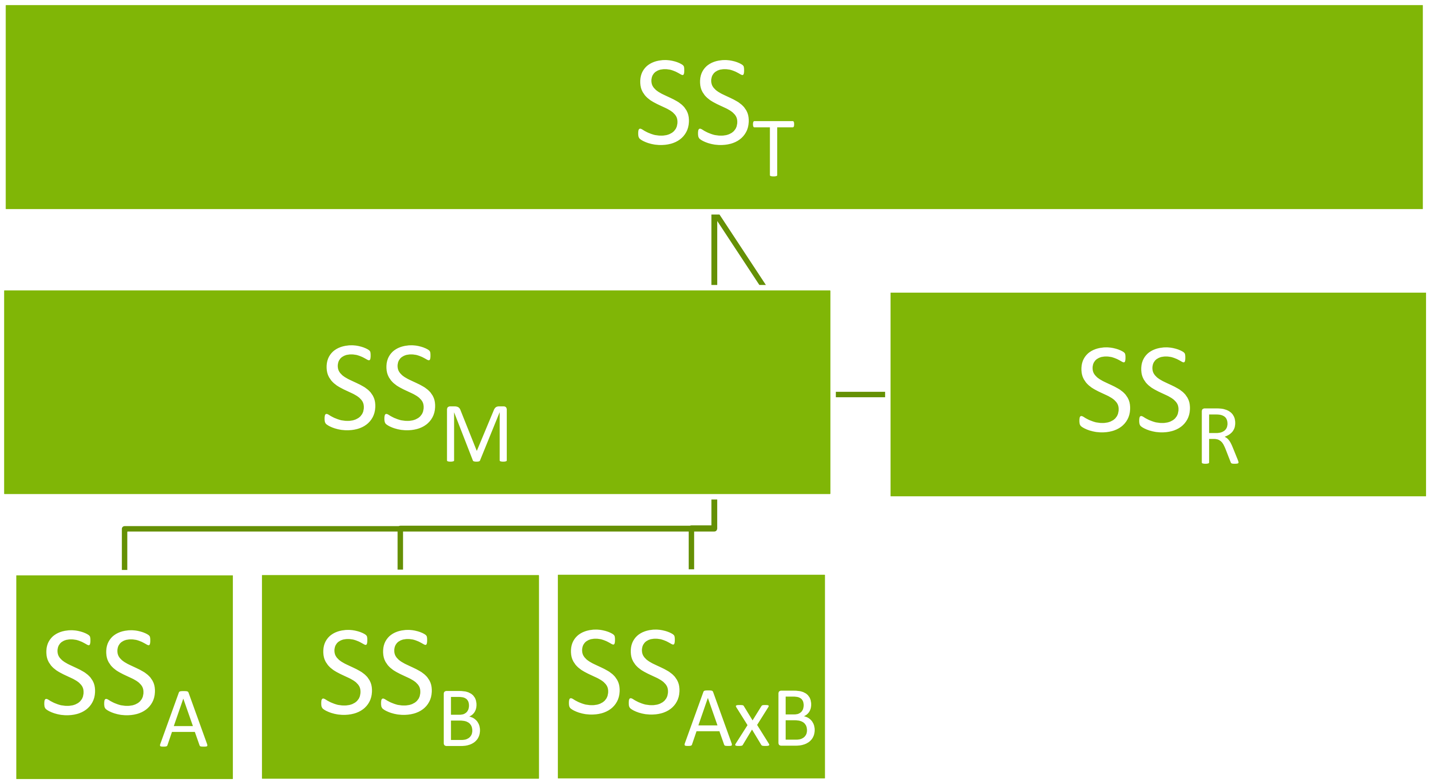
SS_T = Sums of Squares Total

SS_M = Sums of Squares Model

(Systematic Variance)

SS_R = Sums of Squares Error

(Unsystematic Variance or error)



SS_T
SS_M
SS_R
SS_A
SS_B
SS_AxB

Two-way Independent-groups ANOVA: Steps in the Analysis

-- Three F tests required: one for each factor (Factor A; Factor B), and a third for the interaction (A x B).

-- The numerator differs for the three *F* tests, but the denominator for all three is the MS_R.

-- Calculate the sum of squares for factor A (SS_A), factor B (SS_B), the interaction (SS_{AxB}), and the error term (SS_R).

-- Convert each factor's SS to the average sums of square or "mean squares" (MS) by dividing by the appropriate degrees of freedom.

$$F_A = \frac{MS_A}{MS_R} \qquad F_B = \frac{MS_B}{MS_R} \qquad F_{AxB} = \frac{MS_{AxB}}{MS_R}$$

Computing F

SS_T = total squared difference between each score and the grand mean

SS_M = total squared difference between the mean of each subgroup and the grand mean N times

- SS_A = total squared difference between the mean of each group in Factor A (male, female) and the grand mean N times

- SS_B = total squared difference between the mean of each group in Factor B (no body decorations, piercings, tattoes, both) and the grand mean N times

- $SS_{AxB} = SS_M - SS_A - SS_B$

SS_R = total squared difference between each score and the mean of the subgroup the score comes from (male with piercings, male with tattoos, female with piercings, female with tattoos, etc.)

Divide by degrees of freedom to get MS_A, MS_B, MS_{AxB}

	Mean squares	Degrees of freedom
Main effect of A	MS_A	k -1
Main effect of B	MS_B	q - 1
A x B interaction	MS_{AxB}	(k - 1)(q - 1)

Calculating Effect Size:

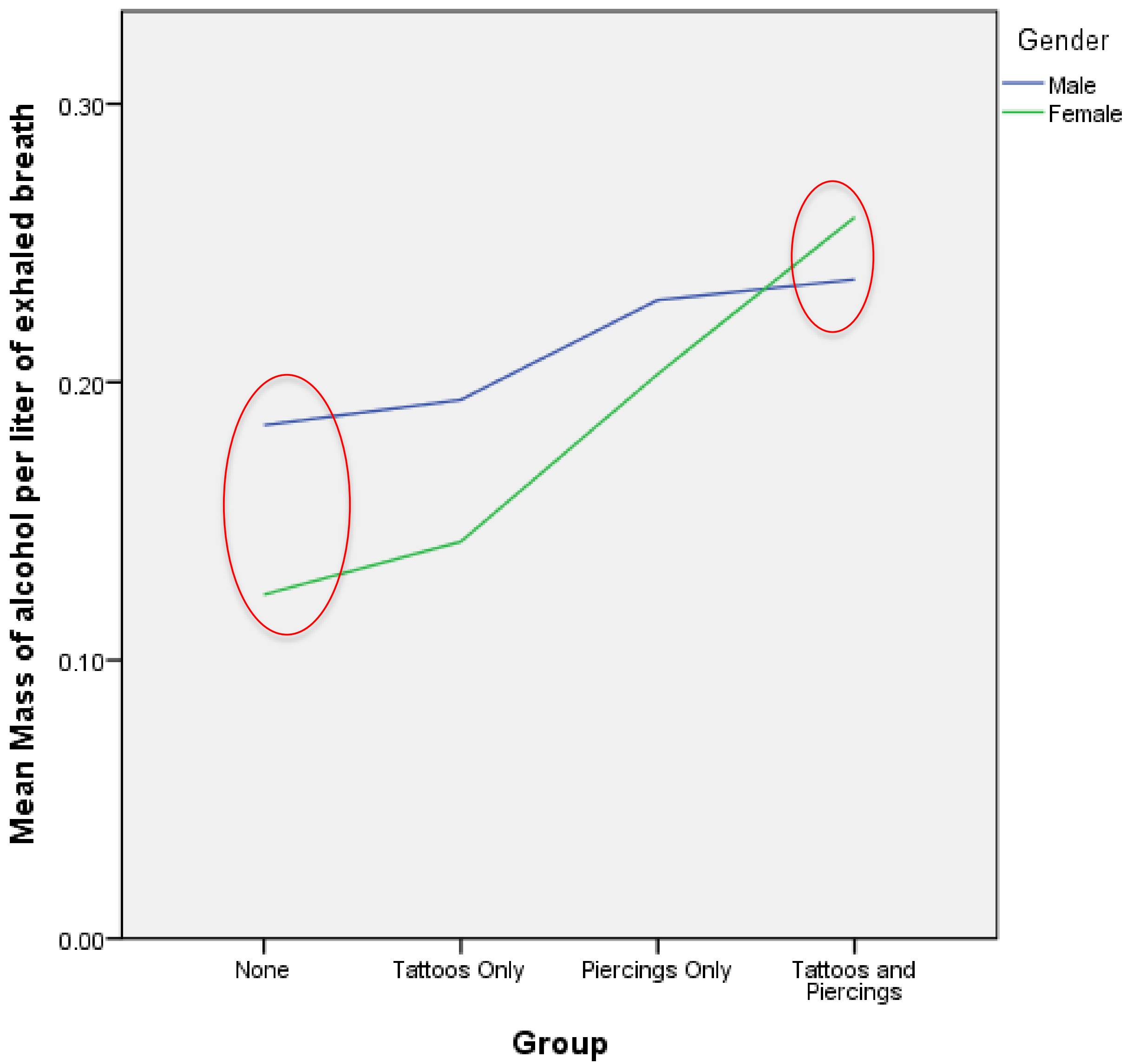
Two-way Independent-groups ANOVA - *Eta* and ω^2:
separately for each effect

$$\omega^2_{\text{effect}} = \frac{SS_M - (df_M \times MS_R)}{SS_T + MS_R}$$

ω^2_A ω^2_B ω^2_{AxB}

(Partial) Eta^2: SS_M/SS_T

- Omnibus ANOVA
  - initial test of main effects and interaction effects
- If main effect is significant and there are more than two levels, conduct post-hoc tests or planned comparisons
- If interaction effect is statistically significant,
  - Simple main effects can be looked at:
    - difference between men with no body decorations and women with no body decorations
    - Difference between men with both piercings and tattooes and women with both piercings and tattoos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0.30
0.20
0.10
0.00
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group
Gender
Male
Female

Running Factorial ANOVA in SPSS

Univariate because there is only 1 **dependent** variable

- Analyze > General Linear Model > Univariate
  - Options: descriptives, effect sizes and estimated marginal means for factors + pairwise comparisons for estimated marginal means
    - Estimated marginal means: population estimates calculated by adjusting the observed means for covariates
  - Contrasts (for factors with more than two levels):
    - Deviation. Compares the mean of each level (except a reference category) to the mean of all of the levels (grand mean). The levels of the factor can be in any order.
    - Simple. Compares the mean of each level to the mean of a specified level. You can choose the first or last category as the reference.
    - Difference. Compares the mean of each level (except the first) to the mean of previous levels.
    - Helmert. Compares the mean of each level of the factor (except the last) to the mean of subsequent levels.
    - Repeated. Compares the mean of each level (except the last) to the mean of the subsequent level.
    - Polynomial. Compares the linear effect, quadratic effect, cubic effect, and so on.

Between-Subjects Factors

		Value Label	N
Gender	0	Male	1139
	1	Female	826
Group	0	None	1440
	1	Tattoos Only	177
	2	Piercings Only	236
	3	Tattoos and Piercings	112

Descriptive Statistics

Dependent Variable: Mass of alcohol per liter of exhaled breath

Gender	Group	Mean	Std. Deviation	N
Male	None	.1845	.14710	903
	Tattoos Only	.1936	.12657	53
	Piercings Only	.2295	.10340	98
	Tattoos and Piercings	.2368	.16156	85
	Total	.1927	.14504	1139
Female	None	.1236	.07739	537
	Tattoos Only	.1427	.09750	124
	Piercings Only	.2029	.14553	138
	Tattoos and Piercings	.2593	.21027	27
	Total	.1442	.10726	826
Total	None	.1618	.12907	1440
	Tattoos Only	.1579	.10919	177
	Piercings Only	.2139	.13012	236
	Tattoos and Piercings	.2422	.17379	112
	Total	.1723	.13265	1965

Main effect of Gender (SS_A)

Tests of Between-Subjects Effects

Dependent Variable: Mass of alcohol per liter of exhaled breath

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	2.547[a]	7	.364	22.250	.000	.074
Intercept	25.726	1	25.726	1572.850	.000	.446
Gender	.140	1	.140	8.547	.004	.004
BodyDecorations	1.319	3	.440	26.883	.000	.040
Gender * BodyDecorations	.177	3	.059	3.607	.013	.005
Error	32.010	1957	.016			
Total	92.879	1965				
Corrected Total	34.557	1964				

a. R Squared = .074 (Adjusted R Squared = .070)

Main effect of Body Decoration (SS_B)

Effect sizes

Interaction (SS_{AxB})

Contrast Results (K Matrix)

Group Polynomial Contrast[a]			Dependent Variable: Mass of alcohol per liter of exhaled breath
Linear	Contrast Estimate		.074
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		.074
	Std. Error		.010
	Sig.		.000
	95% Confidence Interval for Difference	Lower Bound	.054
		Upper Bound	.094
Quadratic	Contrast Estimate		.009
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		.009
	Std. Error		.010
	Sig.		.370
	95% Confidence Interval for Difference	Lower Bound	-.011
		Upper Bound	.028
Cubic	Contrast Estimate		-.011
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-.011
	Std. Error		.010
	Sig.		.243
	95% Confidence Interval for Difference	Lower Bound	-.030
		Upper Bound	.008

a. Metric = 1.000, 2.000, 3.000, 4.000

Linear contrast is significant

Writing it up

Men and women were categorized into four groups depending on the body decorations they wore. See Table 1 for sample sizes. Their alcohol level was measured with a breathalyzer as they were leaving a pub.

	No decorations	Piercings	Tattoos	Piercings and tattoos
Male	903	98	53	85
Female	537	138	124	27

Table 1. Number of participants in each group.

A 2 x 4 ANOVA with Gender and Body Decoration as between-subject factors revealed a main effect of Gender. On average, men had a higher level of alcohol (M = .19, SD = .14) than women (M = .14, SD = .11), $F(1, 1957) = 8.55$, $p = .004$, $\text{Eta}_p^2 = .004$)

We also found a main effect of Body Decoration. Linear contrast analysis revealed that on average, the more body decorations someone had, the higher their alcohol level was ($F(3, 1957) = 26.88$, $p < .001$, $\text{Eta}_p^2 = .04$)

The Gender x Body Decoration interaction was also statistically significant, $F(3, 1957) = 3.67$, $p = .013$. While women, on average, had lower alcohol levels than men, this difference was reversed for people with both tattoos and piercings. In this group, women had higher levels of alcohol (M = .26, SD = .21) than men (M = .24, SD = .16).

Exercise

- Classdata.sav

Does going out affect men’s and women’s alcohol consumption differently?

Dependent variable: alcohol consumption

Factors: Gender and Going out

(Note: we really should not use GLM here because alcohol consumption is not at all normally distributed. Try a Generalized Linear Model)

Homework: davey(2003).sav

Do our mood and our attitudes influence the way we do our jobs? People were put into a positive, negative or neutral mood and were asked to list things they should check before going on holiday. Half of each mood group was told that they should do their best to list everything they could think of. The other half were told that they should carry on writing their list as long as they liked. The number of items on the list was counted.

- Run descriptives
- Run analysis
- Write up the results
- Include graph of means and CIs