Two or more independent variables

Factorial ANOVA

Outline

- Definitions and Concepts

- Introduction to factorial ANOVA
  - Two-way Independent-groups ANOVA

- Exercises

Definitions

- **Factorial ANOVA**
  - An ANOVA with more than one discrete IV and one scale DV. Each IV is called a factor.
- **Main Effect**
  - The omnibus test of a single factor ignoring any other factors is called a main effect.
- **Interaction**
  - Test of the interaction between the two IVs
  - A relationship involving three variables in which the relationship between two of the variables differs depending upon the third variable. An interaction effect exists when the relationship between two variables changes for different values of a third variable.

Main effect

- Main effects
  - Overall effect of an IV in a factorial design
    - effect on DV *as if* only that IV was studied
- The main effect of a factor is the effect that changing the levels of that factor has on dependent variable scores while ignoring all other factors in the study
- We collapse across a factor. Collapsing across a factor means averaging together all scores from all levels of that factor.

Main Effect: An outcome that represents a consistent difference between levels of a factor

SS_{M1} = difference between the means of each level of Factor1 and the grand mean

SS_{M2} = difference between the means of each level of Factor2 and the grand mean

Interaction

- Interaction effects
  - Combined effect of IVs considered simultaneously
  - An interaction effect occurs when the effect of an independent variable differs depending on the level of a 2nd independent variable.
- The interaction of two factors is called a two-way interaction
  - The two-way interaction effect is the influence on scores that results from combining the levels of factor A with the levels of factor B
  - When you look for the interaction effect, you compare the cell means. When you look for a main effect, you compare the level means.

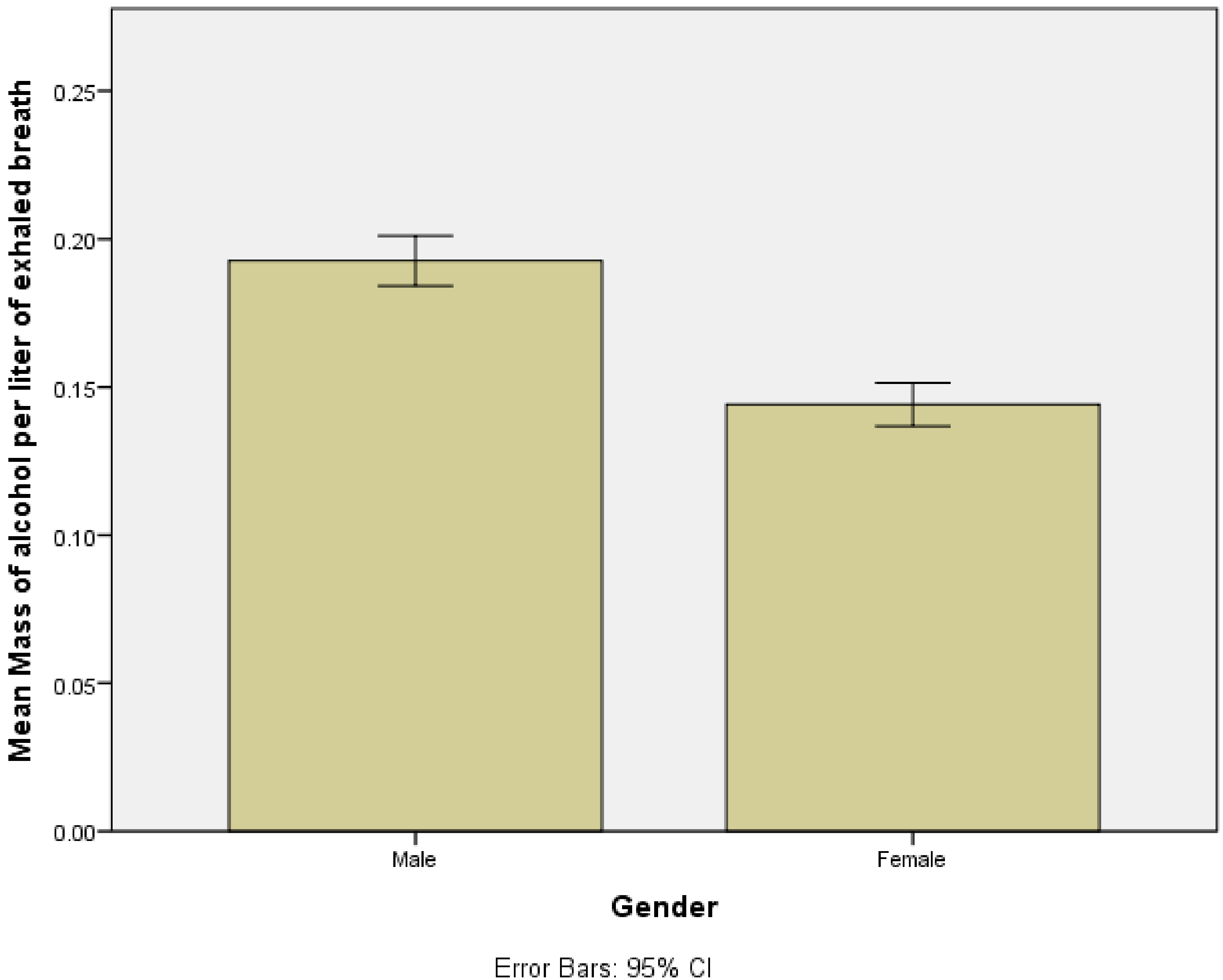
Example: Gueguen(2012).sav

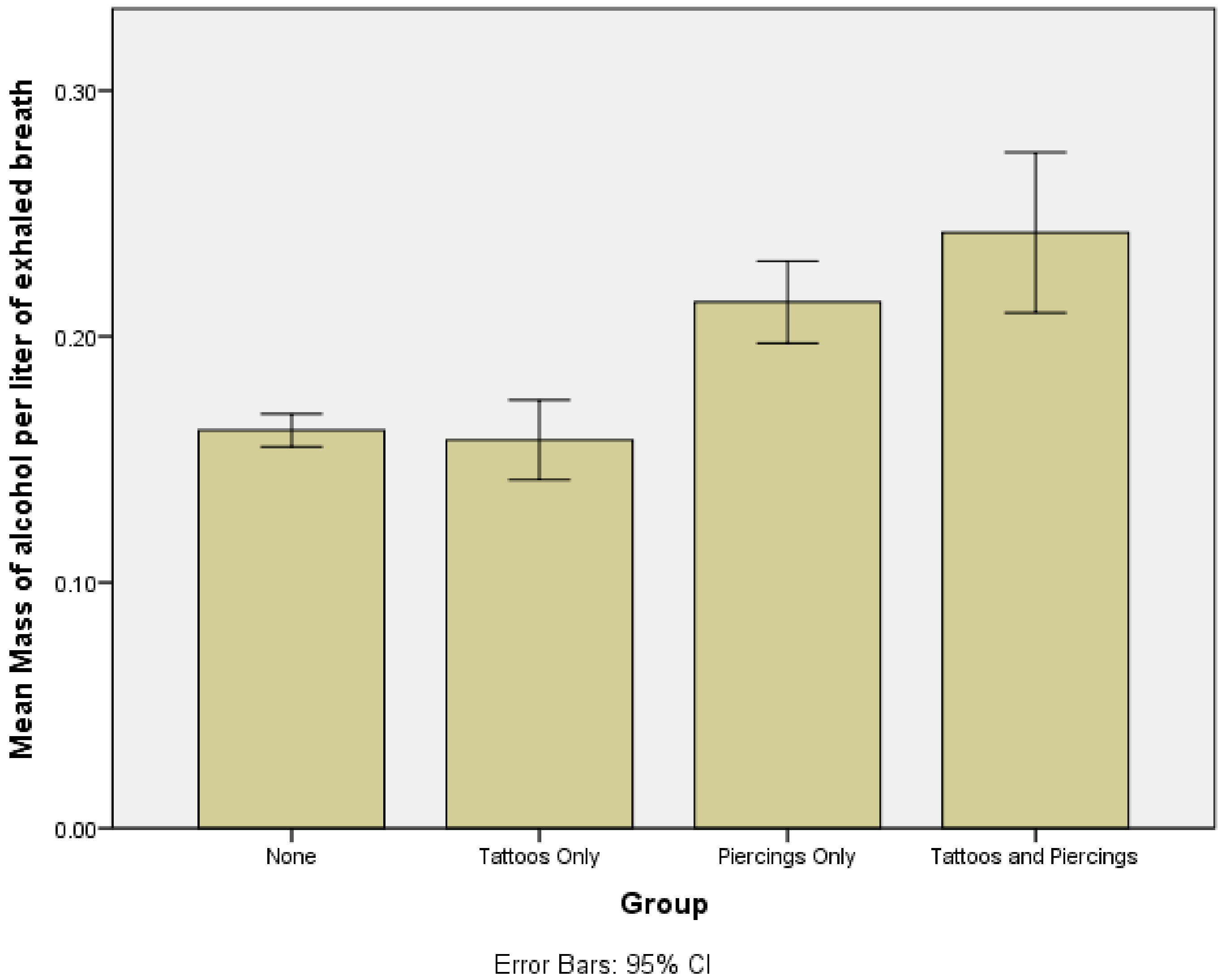
Is risk taking behaviour related to tattoos and body piercings? Does this relationship depend on gender?

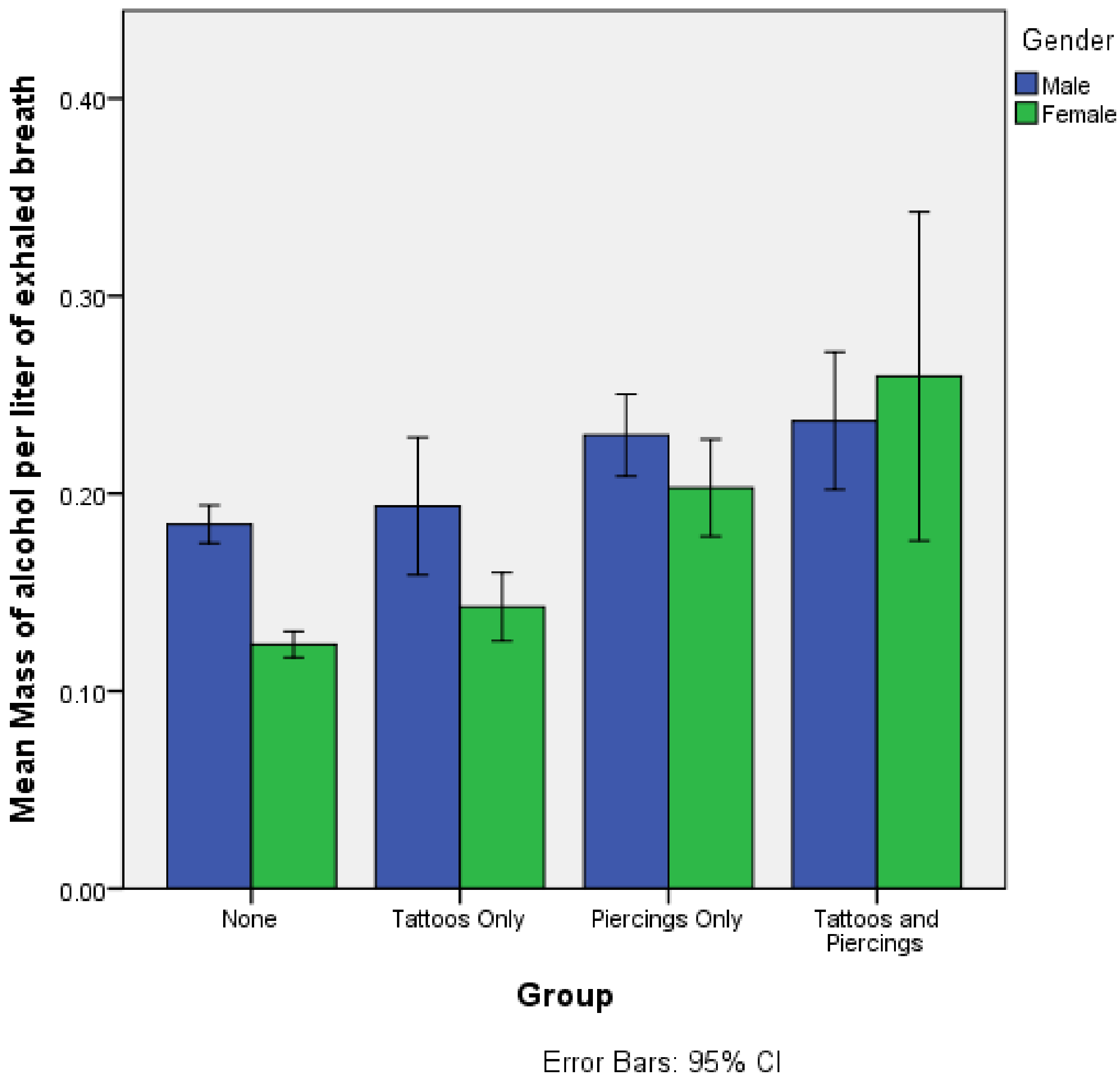
- gender
  - Men and women
- Piercings
  - None, tattoos, piercings, tattoos and piercings
- Blook alcohol level measured when coming out of a bar

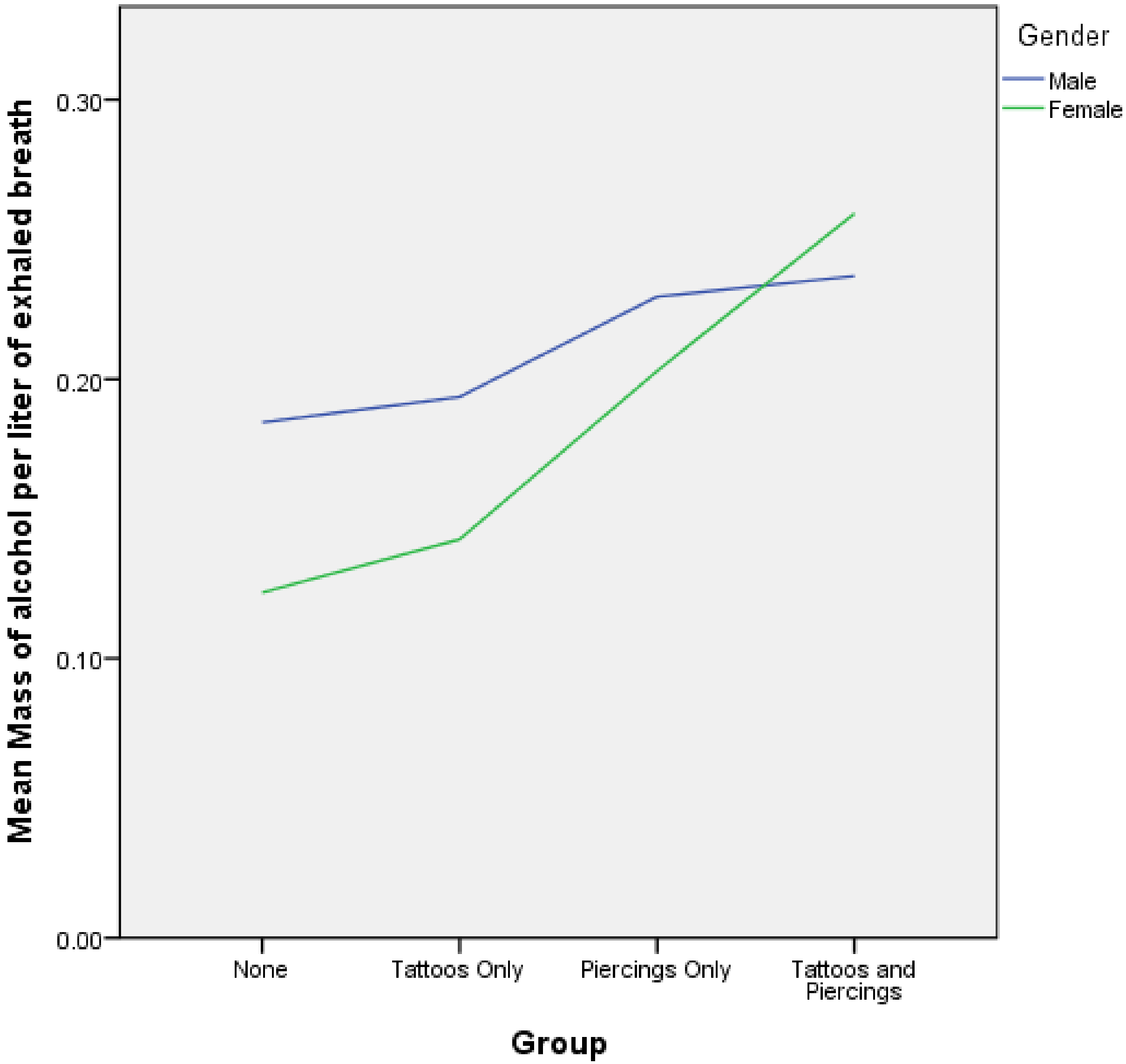
Example

- Factorial combination of 2 IVs
  - Gender
  - Body decoration
  - Factorial combination: 8 conditions
  - Referred to as a 2 x 4 (first IV with two levels, second IV with four levels)
- What we can look at:
  - Effect of Factor A (main effect of Gender)
  - Effect of Factor B (main effect of Body Decoration)
  - Interaction
    - Does the effect of one factor depend on the level of the other factor? Does the effect of body decorations depend on gender?


Mean Mass of alcohol per liter of exhaled breath
0.25
0.20
0.15
0.10
0.05
0.00
Male
Female
Gender
Error Bars: 95% CI


Mean Mass of alcohol per liter of exhaled breath
0.30
0.20
0.10
0.00
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group
Error Bars: 95% CI

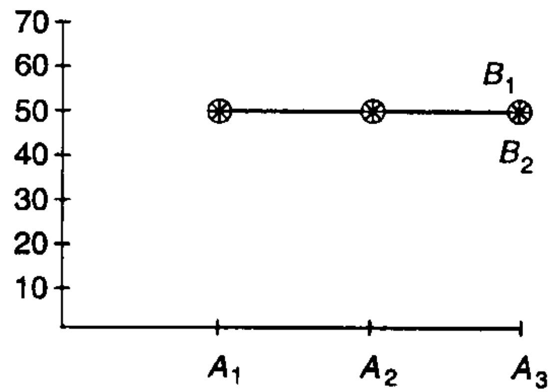

Mean Mass of alcohol per liter of exhaled breath
0.40
0.30
0.20
0.10
0.00
Gender
Male
Female
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group
Error Bars: 95% CI



Gender
Male
Female
0.30
0.20
0.10
0.00
Mean Mass of alcohol per liter of exhaled breath
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group

Possible Outcomes

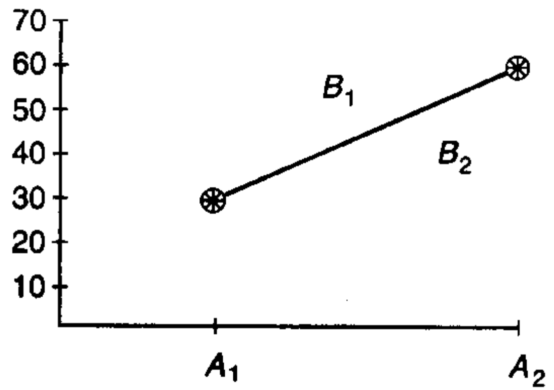
	A_1	A_2	A_3	Mean
B_1	50	50	50	50
B_2	50	50	50	50
Mean	50	50	50	



No effects

(a) 3 × 2 Factorial (*A*, *B*, and the interaction are not significant)

	A_1	A_2	Mean
B_1	30	60	45
B_2	30	60	45
Mean	30	60	

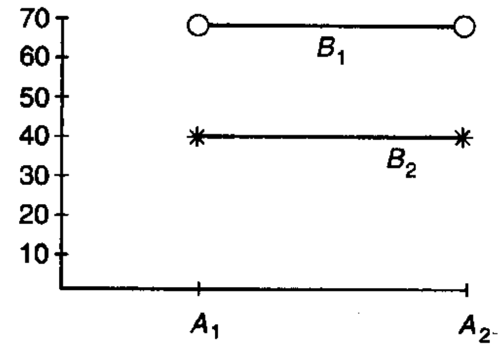


Main effect of A

(b) 2 × 2 Factorial (*A* is significant; *B* and the interaction are not significant)

Possible Outcomes

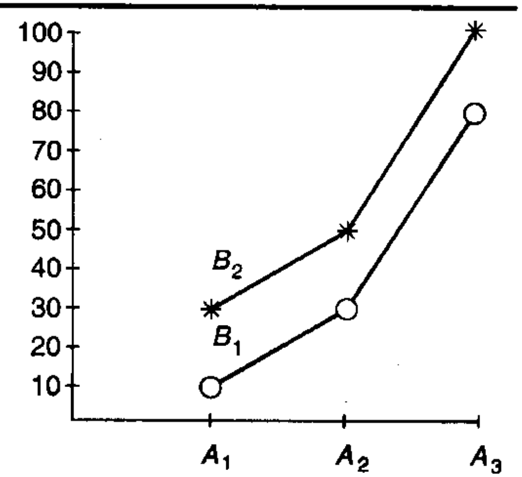
	A_1	A_2	Mean
B_1	70	70	70
B_2	40	40	40
Mean	55	55	



(c) 2 × 2 Factorial (B is significant; A and the interaction are not significant)

Main effect of B

	A_1	A_2	A_3	Mean
B_1	10	30	80	40
B_2	30	50	100	60
Mean	20	40	90	

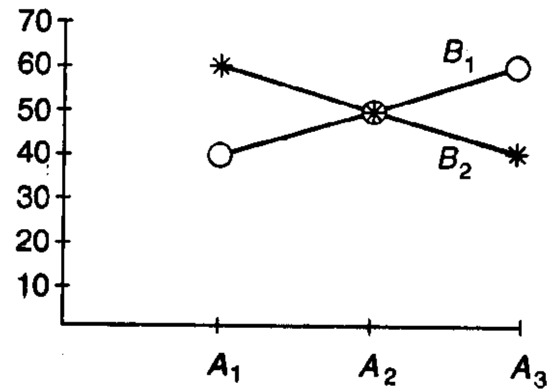


(d) 3 × 2 Factorial (A and B are significant; the interaction is not significant)

Main effect of A and B

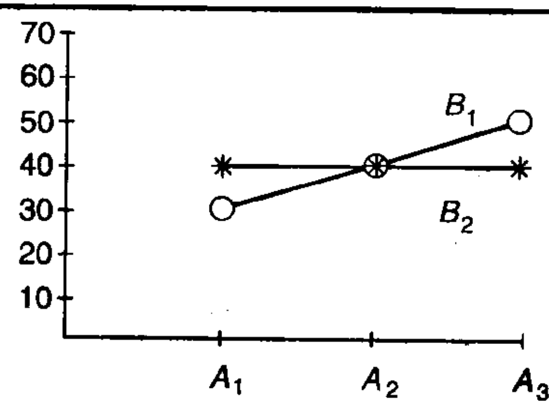
Possible Outcomes

	A_1	A_2	A_3	Mean
B_1	40	50	60	50
B_2	60	50	40	50
Mean	50	50	50	



(e) 3 × 2 Factorial (the interaction is significant: *A* and *B* are not significant)

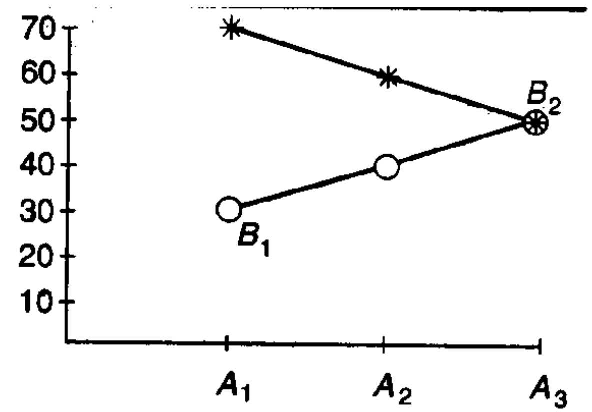
	A_1	A_2	A_3	Mean
B_1	30	40	50	40
B_2	40	40	40	40
Mean	35	40	45	



(f) 3 × 2 Factorial (*A* and the interaction are significant; *B* is not significant)

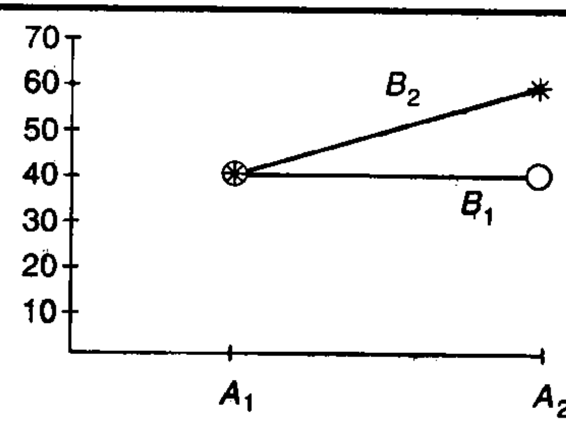
Possible Outcomes

	A_1	A_2	A_3	Mean
B_1	30	40	50	40
B_2	70	60	50	60
Mean	50	50	50	



(g) 3 × 2 Factorial (*B* and the interaction are significant; *A* is not significant)

	A_1	A_2	Mean
B_1	40	40	40
B_2	40	60	50
Mean	40	50	



(h) 2 × 2 Factorial (*A*, *B*, and the interaction are significant)

Factorial ANOVA

- ANOVA can analyze any factorial design
- The number of effects will depend on the number of independent variables (IVs)
  - 2 IVs: A & B main effects; AB interaction
  - 3 IVs: A, B, & C main effects; AB, AC, BC, & ABC interactions
  - 4 IVs: A, B, C, & D main effects; AB, AC, AD, BC, BD, CD, ABC, ABD, BCD, & ABCD interactions
  - Etc.

FORMALLY

Assumptions

The two-way independent groups ANOVA test requires the following statistical assumptions:

1. Random and independent sampling.

2. Data are from normally distributed populations.
   Note: This test is robust against violation of this assumption if $n \geq 30$ for all groups.

3. Variances in these populations are roughly equal (Homogeneity of variance).
   Note: This test is robust against violation of this assumption if all group sizes are equal.

The maths behind ANOVA: A Quick Review

Analysis of variance (ANOVA) is a method of testing the null hypothesis that three or more means are roughly equal.

ANOVA produces a test statistic termed

the *F-ratio*.

$$\frac{\text{Systematic Variance } (SS_M)}{\text{Unsystematic Variance } (SS_R)}$$

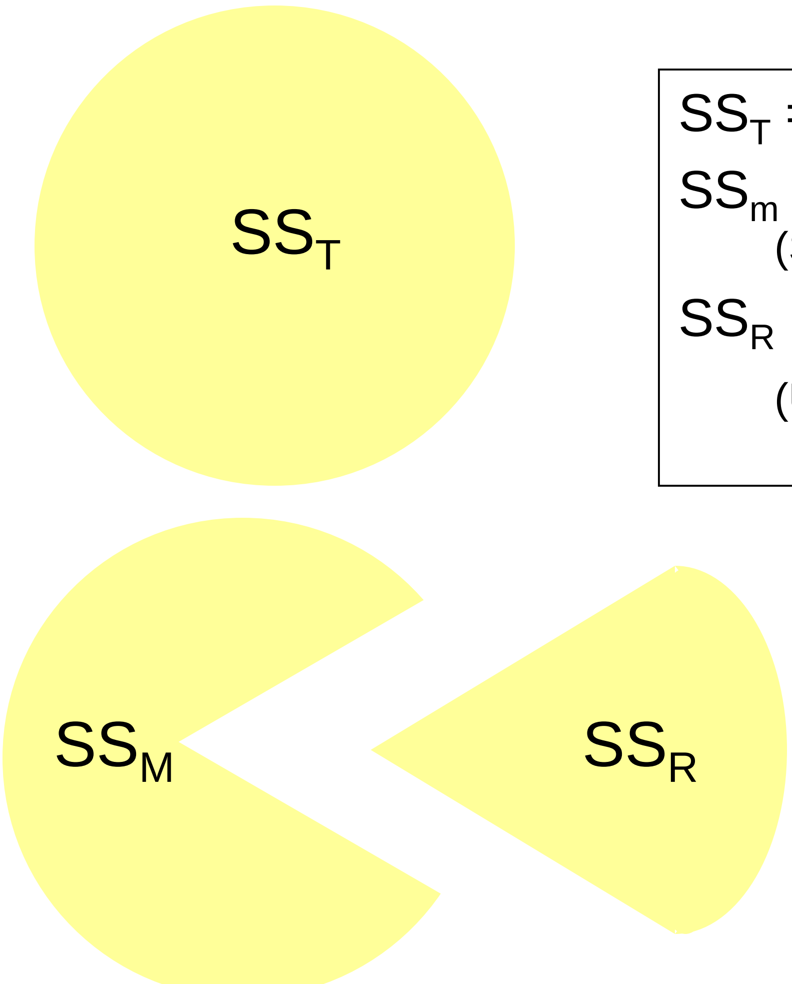
The F-ratio tells us only that the experimental manipulation has had an effect—not where the effect has occurred.

-- Planned comparisons

-- Post-hoc tests

-- Purpose of follow-up tests? Control Type I error rate at 5%.

Separating Out The Variance



SS_T = Sums of Squares Total

SS_m = Sums of Squares Model
(Systematic Variance)

SS_R = Sums of Squares Error
(Unsystematic Variance or error)

Two-way Independent-groups ANOVA: Steps in the Analysis

-- Three F tests required: one for each factor (Factor A; Factor B), and a third for the interaction (A x B).

-- The numerator differs for the three *F* tests, but the denominator for all three is the MS_R.

-- Calculate the sum of squares for factor A (SS_A), factor B (SS_B), the interaction (SS_{AxB}), and the error term (SS_R).

-- Convert each factor's SS to the average sums of square or "mean squares" (MS) by dividing by the appropriate degrees of freedom).

$$F_A = \frac{MS_A}{MS_R} \qquad F_B = \frac{MS_B}{MS_R} \qquad F_{AxB} = \frac{MS_{AxB}}{MS_R}$$

Computing F

SS_T = total squared difference between each score and the grand mean

SS_M = total squared difference between the mean of each subgroup (Grade 1 with Barbie, Grade 1 with Emme, etc.) and the grand mean N times

- SS_A = total squared difference between the mean of each group in Factor A (male, female) and the grand mean N times
- SS_B = total squared difference between the mean of each group in Factor B (no body decorations, piercings, tattoes, both) and the grand mean N times
- $SS_{AxB} = SS_M - SS_A - SS_B$

SS_R = total squared difference each score and the mean of the subgroup the score comes from (male with piercings, male with tattoos, female with piercings, female with tattoos, etc.)

Divide by degrees of freedom to get MS_A, MS_B, MS_{AxB},

	Mean squares	Degrees of freedom
Main effect of A	MS_A	k -1
Main effect of B	MS_B	q - 1
A x B interaction	MS_{AxB}	(k - 1)(q - 1)

Calculating Effect Size:

Two-way Independent-groups ANOVA - *Eta* and ω^2:

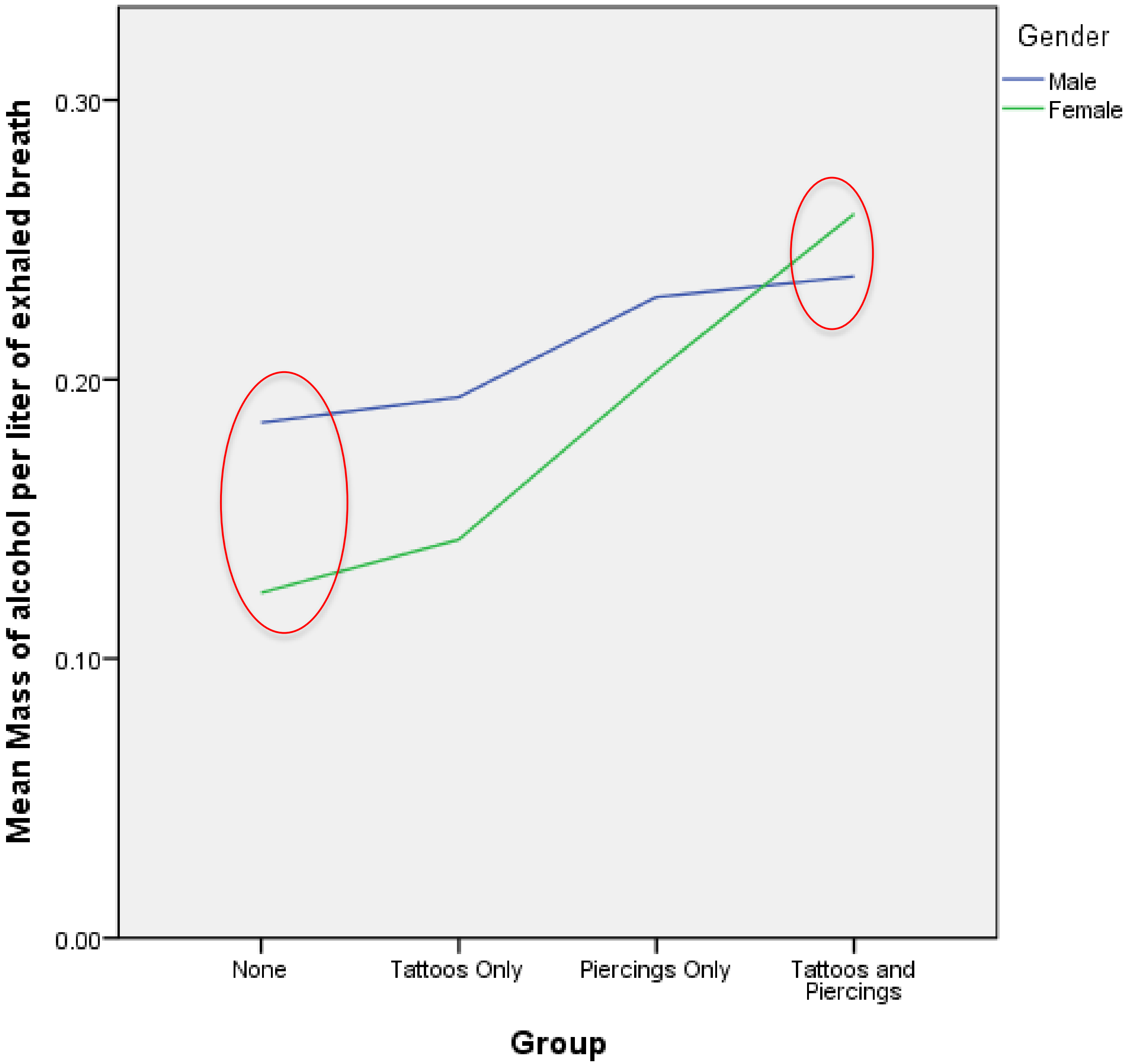
separately for each effect

$$\omega^2_{\text{effect}} = \frac{SS_M - (df_M \times MS_R)}{SS_T + MS_R}$$

ω^2_A ω^2_B ω^2_{AxB}

(Partial) Eta^2: SS_M/SS_T

- Omnibus ANOVA
  - initial test of main effects and interaction effects
- If main effect is significant and there are more than two levels, conduct post-hoc tests or planned comparisons
- If interaction effect is statistically significant,
  - Simple main effects can be looked at:
    - difference between men with no body decorations and women with no body decorations
    - Difference between men with both piercings and tattooes and women with both piercings and tattoos



Gender
Male
Female
Mean Mass of alcohol per liter of exhaled breath
0.30
0.20
0.10
0.00
None
Tattoos Only
Piercings Only
Tattoos and Piercings
Group

Men and women were categorized into four groups depending on the body decorations they wore. See Table 1 for sample sizes. Their alcohol level was measured with the breathalyzer as they were leaving a pub.

	No decorations	Piercings	Tattoos	Piercings and tattoos
Male	903	98	53	85
Female	537	138	124	27

Table 1. Number of participants in each group.

There was a main effect of Gender. On average, men had a higher level of alcohol (M = .211, SE = .007) than women (M = .182, SE = .007), $F(1, 1957) = 8.55$, $p = .004$.

We also found a main effect of Body Decoration. On average, the more body decorations someone had, the higher their alcohol level was $F(3, 1957) = 26.88$, $p < .001$.

The Gedner x Body Decoration interaction was also statistically significant, $F(3, 1957) = 3.67$, $p = .013$. While women, on average, had lower alcohol levels than men, this difference was reversed for people with both tattoos and piercings. In this group, women had higher levels of alcohol (M = .259, SE = .025) than men (M = .237, SE = .014).

Homework: davey(2003).sav

Do our mood and our attitudes influence the way we do our jobs? People were put into a positive, negative or neutral mood and were asked to list things they should check before going on holiday. Half of each mood group was told that they should do their best list everything they could think of. The other half were told that they should carry on writing their list as long as they liked. The number of items on the list was counted.

- Run descriptives
- Run analysis
- Write up the results
- Include graph of means and CIs