Numerical Cognition Without Words: Evidence from Amazonia

Peter Gordon

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Members of the Pirahã tribe use a "one-two-many" system of counting. I ask whether speakers of this innumerate language can appreciate larger numerosities without the benefit of words to encode them. This addresses the classic Whorfian question about whether language can determine thought. Results of numerical tasks with varying cognitive demands show that numerical cognition is clearly affected by the lack of a counting system in the language. Performance with quantities greater than three was remarkably poor, but showed a constant coefficient of variation, which is suggestive of an analog estimation process.

Is it possible that there are some concepts that we cannot entertain because of the language that we speak? At issue here is the strongest version of Benjamin Lee Whorf's hypothesis that language can determine the nature and content of thought. The strong version of Whorf's hypothesis goes beyond the weaker claim that linguistic structure simply influences the way that we think about things in our everyday encounters. For example, recent studies suggest that language might affect how people mentally encode spatial relations (*1*–*3*), and how they conceive of the nature of individual objects and their material substances (*4*). However, none of these studies suggest that linguistic structure prevents us from entertaining the concepts that are available to speakers of alternative linguistic systems.

The question of whether linguistic determinism exists in the stronger sense has two parts. The first is whether languages can be incommensurate: Are there terms that exist in one language that cannot be translated into another? The second is whether the lack of such translation precludes the speakers of one language from entertaining concepts that are encoded by the words or grammar of the other language. For many years, the answer to both questions appeared to be negative. Although languages might have different ways in which situations are habitually described, it has generally been accepted that there would always be some way in which one could capture the equivalent meaning in any other language (*5*). Of course, when speaking of translatable concepts, we do not mean terms like "molecule" or "quark," which would not exist in a culture without advanced scientific institutions. Failure to know what molecules or quarks are does not signal an inability to understand the English language—surely people were still speaking English before such terms were introduced. On the other hand, one would question someone's command of English if they did not understand the basic vocabulary and grammar.

Department of Biobehavioral Sciences, Columbia University, 525 West 120th Street, New York, NY 10027, USA. E-mail: pgordon@tc.columbia.edu

Words that indicate numerical quantities are clearly among the basic vocabulary of a language like English. But not all languages contain fully elaborated counting systems. Although no language has been recorded that completely lacks number words, there is a considerable range of counting systems that exists across cultures. Some cultures use a finite number of body parts to count 20 or 30 body tags (*6*). Many cultures use particular body parts like fingers as a recursive base for the count system as in our 10-based system. Finally, there are cultures that base their counting systems on a small number between 2 and 4. Sometimes, the use of a small-number base is recursive and potentially infinite. For example, it is claimed that the Gumulgal South Sea Islanders counted with a recursive binary system: 1, 2, 2'1, 2'2, 2'2'1, and so on (*6*).

The counting system that differs perhaps most from our own is the "one-two-many" system, where quantities beyond two are not counted but are simply referred to as "many." If a culture is limited to such a counting system, is it possible for its members to perceive or conceptualize quantities beyond the limited sets picked out by the counting sequence, or to make what we consider to be quite trivial distinctions such as that between four versus five objects? The Pirahã are such a culture. They live along the banks of the Maici River in the Lowland Amazonia region of Brazil. They maintain a predominantly hunter-gatherer existence and reject assimilation into mainstream Brazilian culture. Almost completely monolingual in their own language, they have a population of less than 200 living in small villages of 10 to 20 people. They have only limited exchanges with outsiders, using primitive pidgin systems for communicating in trading goods without monetary exchange and without the use of Portuguese count words. The Pirahã counting system consists of the words: "hói" (falling tone = "one") and "hoí" (rising tone = "two"). Larger quantities are designated as "baagi" or "aibai" (= "many").

I was able to take three field trips, ranging from 1 week to 2 months, living with the Pirahã along with Daniel Everett and Keren Everett, two linguists who have lived and worked with the tribe for over 20 years and are completely familiar with their language and cultural practices. Observations were informed by their background of continuous and extensive immersion in the Pirahã culture. During my visits, I became interested in the counting system of the Pirahã that I had heard about and wanted to examine whether they really did have only two numbers and how this would affect their ability to perceive numerosities that extended beyond the limited count sequence.

Year 1: Initial observations. On my first week-long trip to the two most up-river Maici villages, I began with informal observations of the Pirahã use of the number words for one and two. I was also interested in the possibility that the one-two-many system might actually be a recursive base-2 system, that their limited number words might be supplemented by more extensive finger counting, or that there might be taboos associated with counting certain kinds of objects as suggested by Zaslavsky in her studies of African counting systems (*7*, *8*). Keren Everett developed some simple tasks to see if our two Pirahã informants could refer to numerosities of arrays of objects using Pirahã terms and any finger counting system they might have. Instructions and interactions with participants were in the Pirahã language. When it was necessary to refer to the numerosity of an array, Keren

Table 1. Use of fingers and number words by Pirahã participant. The arrow (→) indicates a shift from one quantity to the next.

No. of objects	Number word used	No. of fingers
1	hói (= 1)	
2	hoí (= 2)	2
	aibaagi (= many)	
3	hoí (= 2)	3
4	hoí (= 2)	5 → 3
	aibai (= many)	
5	aibaagi (= many)	5
6	aibaagi (= many)	6 → 7
7	hói (= 1)*	1
	aibaagi (= many)	5 → 8
8		5 → 8 → 10
9	aibaagi (= many)	5 → 10
10		5

*This use of "one" might have been a reference to adding one rather than to the whole set of objects.

Everett used the Portuguese number words embedded in Pirahã dialogue. Such terms are understood by the Pirahã to be the language of Brazilians, but their meaning is not understood. In addition to this short session, during the first year trip, I continuously took opportunities to probe for counting abilities in everyday situations.

The outcome of these informal studies revealed the following: (i) There was no recursive use of the count system—the Pirahã never used the count words in combinations like "hói-hoí" to designate larger quantities. (ii) Fingers were used to supplement oral enumeration, but this was highly inaccurate even for small numbers less than five. In addition, "hói" and "hoí," the words for "one" and "two," were not always used to denote those quantities. Whereas the word for "two" always denoted a larger quantity than the word for "one" (when used in the same context), the word for "one" was sometimes used to denote just a small quantity such as two or three or sometimes more. An example of the use of counting words and finger counting is given in Table 1 in one of the informal sessions with an informant who appeared to be in his 50s. Videotaped extracts from the session are included in the supporting online materials (movie S1).

The interpretation of these observations is limited by their informal nature and small sample size. However, the observations are supplemented with 20 years of observation by the Everetts as trained linguists in their analysis of the Pirahã language. One particularly interesting finding is that "hói" appears to designate "roughly one"—or a small quantity whose prototype is one. Most of the time, in the enumeration task, "hói" referred to one, but not always. An analogy might be when we ask for "a couple of *X*s" in English, where the prototypical quantity is two, but we are not upset if we are given three or four objects. However, we surely would be upset if given only one object, because the designation of a single object has a privileged status in our language. There is no concept of "roughly one" in a true integer system. Even the informal use of the indefinite article "a *X*" strictly requires a singular reference. In Pirahã, "hói" can also mean "small," which contrasts with "ogii" (= big), suggesting that the distinction between discrete and continuous quantification is quite fuzzy in the Pirahã language.

Year 2: Experiments in nonverbal numerical reasoning. On my second visit to the Pirahã villages for a 2-month period, I developed a more systematic set of procedures for evaluating the numerical competence of members of the tribe. The experiments were designed to require some combination of cognitive skills such as the need for memory, speed of encoding, and mental-spatial transformations. This would reveal the extent to which such task demands interact with numerical ability, such as it is. Details of the methods are available on *Science* Online (*9*). There were seven participants, who included all six adult males from two villages and one female. Most of the data were collected on four of the men who were consistently available for participation. The tasks were devised to use objects that were available and familiar to the participants (sticks, nuts, and batteries). The results of the tasks, along with schematic diagrams, are presented in Fig. 1. These are roughly ordered in terms of increasing cognitive demand. Any estimation of a person's numerical competence will always be confounded with performance factors of the task. Because this is unavoidable, it makes sense to explore how performance is affected by a range of increasingly demanding tasks.

In the matching tasks (A, B, C, D, and F), I sat across from the participant and with a stick dividing my side from theirs, I presented an array of objects on my side of the stick (below the line in the figures) and they responded by placing a linear array of AA batteries (5.0 cm by 1.4 cm) on their side of the table (above the line). The matching task provides a kind of concrete substitute for counting. It shares the element

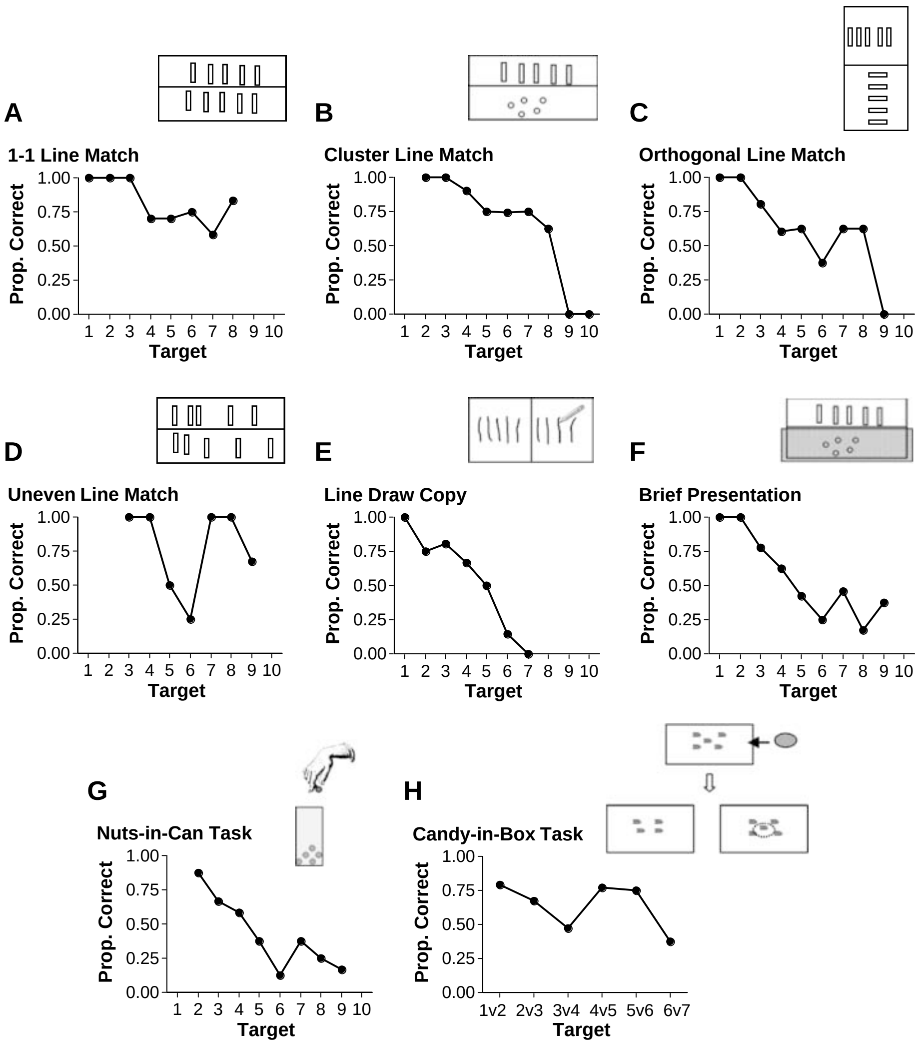

Fig. 1. Results of number tasks with Pirahã villagers (n = 7). Rectangles indicate AA batteries (5.0 cm by 1.4 cm), and circles indicate ground nuts. Center line indicates a stick between the author's example array (below the line) and the participant's attempt to "make it the same" (above the line). Tasks A through D required the participant to match the lower array presented by the author using a line of batteries; task E was similar, but involved the unfamiliar task of copying lines drawn on paper; task F was a matching task where the participant saw the numerical display for only about 1 s before it was hidden behind a screen; task G involved putting nuts into a can and withdrawing them one by one; (participants responded after each withdrawal as to whether the can still contained nuts or was empty); task H involved placing candy inside a box with a number of fish drawn on the lid (this was then hidden and brought out again with another box with one more or one less fish on the lid, and participants had to choose which box contained the candy).

of placing tokens in one-to-one correspondence with individuals in a to-be-counted group. The first matching tasks began with simple linear arrays of batteries. This progressed to clusters of nuts matched to the battery line, orthogonal matching of battery lines, matching of battery lines that were unevenly spaced, and copying lines on a drawing. In all of these matching experiments, participants responded with relatively good accuracy with up to 2 or 3 items, but performance deteriorated considerably beyond that up to 8 to 10 items. In the first simple linear matching task A, performance hovered around 75% up to the largest quantities. Matching tasks with greater cognitive demands required mental transposition of the sample array to the match array without benefit of tagging for numerical quantity. Performance dropped precipitously to 0% for the larger target set sizes in these tasks. One exception was task D with unevenly spaced objects. Although this was designed to be a difficult task, participants showed an anomalous superiority for large numerosities over small. Performance initially deteriorated with increased set size up to 6 items, then shot up to near perfect performance for set size 7 through 10. A likely interpretation of this result was that the uneven spacing for larger set sizes promoted recoding of arrays into smaller configurations of two or three items. This allowed participants to use a chunking strategy of treating each of the subgroups as a matching group.

When time constraints were introduced in task F (exposing the array for only 1 s), performance was drastically affected and there was a clear correlation between set size and accuracy beginning at set size 3. A line-drawing task (E) was highly affected by set size, being one of the worst performances of all. Not only do the Pirahã not count, but they also do not draw. Producing simple straight lines was accomplished only with great effort and concentration, accompanied by heavy sighs and groans. The final two tasks (G and H) required participants to keep track of a numerical quantity through visual displacement. In one case, they were first allowed to inspect an array of nuts for about 8 s. The nuts were placed in a can, and then withdrawn one at a time. Participants were required to say, after each withdrawal, if there were still any nuts left in the can or if it was empty. Performance was predictably strongly affected by set size from the very smallest quantities. The final task involved hiding candy in a box, which had a picture of some number of fish on the lid. The box was then hidden behind the author's back, and two cases were revealed, the original with the candy, and another with one more or one less fish on the lid. For quite small comparisons such as three versus four, performance rarely went over 50% chance responding.

There is a growing consensus in the field of numerical cognition that primitive numerical abilities are of two kinds: First, there is the ability to enumerate accurately small quantities up to about three items, with only minimal processing requirements (*10–16*). I originally termed this ability "parallel individuation" (*17*, *18*), referring to how many items one can encode as discrete unique individuals at the same time in memory. Without overt counting, humans and other animals possess an analog procedure whereby numerical quantities can be estimated with a limited degree of accuracy (*11*, *19–26*). Many researchers believe that large-number estimation, although based on individuated elements, is coalesced into a continuous analog format for mental representation. For example, the discrete elements of a large number array might be represented as a continuous length of a line, where a longer line inexactly represents a larger numerosity.

When people use this analog estimation procedure, the variability of their estimates tends to increase as the target set size increases. The ratio of average error to target set size is known as Weber's fraction and can be indexed by a measure known as the coefficient of variation—the standard deviation of the estimates divided by set size (*23*). Although performance by the Pirahã on the present tasks was quite poor for set sizes above two or three, it was not random. Figure 2 shows the mean response values mapped against the target values for all participants in the simple matching tasks A, B, C, and F. The top graph shows that mean responses and target values are almost identical. This means that the Pirahã participants were trying hard to get the answers correct, and they clearly understood the tasks. The lower graph in Fig. 2 shows that the standard deviation of the estimates increases in proportion to the set size, resulting in a constant coefficient of variation of about 0.15 after set size three, as predicted by the dual model of mental enumeration. This value for the coefficient of variation is about the same as one finds in college students engaged in numerical estimation tasks (*23*). Data for individual tasks and individual participants were consistent with the averaged trends shown in Fig. 2. Graphs are available in the supporting online materials (figs. S2 and S3).

The results of these studies show that the Pirahã's impoverished counting system limits their ability to enumerate exact quantities when set sizes exceed two or three items. For tasks that required additional cognitive processing, performance deteriorated even on set sizes smaller than three. Participants showed evidence of using analog magnitude estimation and, in some cases, they took advantage of spatial chunking to decrease the cognitive demands of larger set sizes. This split between exact enumeration ability for set sizes smaller than three and analog estimation for larger set sizes parallels findings from laboratory experiments with adults who are prevented from explicit counting; studies of numerical abilities in prelinguistic infants, monkeys, birds, and rodents; and in recent studies using brain-imaging techniques (*11*, *23–30*).

The analog estimation abilities exhibited by the Pirahã are a kind of numerical competence that appears to be immune to numerical language deprivation. But because lower animals also exhibit such abilities, robustness in the absence of language is already established. The present experiments allow us to ask whether humans who are not exposed to a number system can represent exact quantities for medium-sized sets of four or five. The answer appears to be negative. The Pirahã inherit just the abilities to exactly enumerate small sets of less than three items if processing factors are not unduly taxing (*31*).

In evaluating the case for linguistic determinism, I suggest that the Pirahã language is incommensurate with languages that have counting systems that enable exact enumeration. Of particular interest is the fact that the Pirahã have no privileged name for the singular quantity. Instead, "hói" meant "roughly one" or "small," which precludes any precise translation of exact numerical terms. The present study represents a rare and perhaps unique case for strong linguistic determinism. The

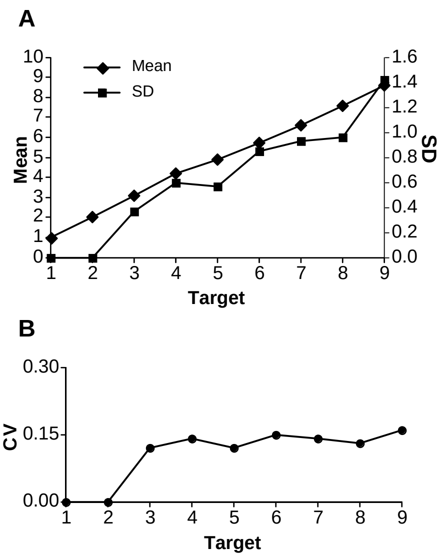

Fig. 2. **(A)** Mean accuracy and standard deviation of responses in matching tasks and **(B)** coefficient of variation. Figures for individual tasks and individual participants are available in the supporting online materials.

study also provides a window into how the possibly innate distinction (*26*) between quantifying small versus large sets of objects is relatively unelaborated in a life without number words to capture those exact magnitudes (*32*).

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31. Cordes *et al*. (*24*) suggest that analog representations exist even for $n = 2$, because subjects made errors on a task in which counting was suppressed during rapid button pressing. However, errors in this range also occurred when subjects counted and might have been the result of perseveration errors rather than reflecting numerical representations.
32. One can safely rule out that the Pirahã are mentally retarded. Their hunting, spatial, categorization, and linguistic skills are remarkable, and they show no clinical signs of retardation.

33. I thank D. Everett and K. Everett for making this research possible; SIL in Porto Velho and E. Ramos for logistical support; and S. Carey, L. Feigenson, D. Everett, C. Tamis-LeMonda, M. Miozzo, G. Marcus, F. Xu, and K. Adolph for comments on the paper.

Supporting Online Material
www.sciencemag.org/cgi/content/full/1094492/DC1
Methods
SOM Text
Figs. S1 to S3
Movies S1 and S2
References

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Exact and Approximate Arithmetic in an Amazonian Indigene Group

Pierre Pica,[1] Cathy Lemer,[2] Véronique Izard,[2] Stanislas Dehaene[2]*

Is calculation possible without language? Or is the human ability for arithmetic dependent on the language faculty? To clarify the relation between language and arithmetic, we studied numerical cognition in speakers of Mundurukú, an Amazonian language with a very small lexicon of number words. Although the Mundurukú lack words for numbers beyond 5, they are able to compare and add large approximate numbers that are far beyond their naming range. However, they fail in exact arithmetic with numbers larger than 4 or 5. Our results imply a distinction between a nonverbal system of number approximation and a language-based counting system for exact number and arithmetic.

All science requires mathematics. The knowledge of mathematical things is almost innate in us.... This is the easiest of sciences, a fact which is obvious in that no one's brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon.

Roger Bacon (1214–1294),
English philosopher and scientist

[1]Unité Mixte de Recherche 7023 "Formal Structures of Language," CNRS and Paris VIII University, Paris, France. [2]Unité INSERM 562 "Cognitive Neuroimaging," Service Hospitalier Frédéric Joliot, CEA/DSV, 91401 Orsay Cedex, France.

*To whom correspondence should be addressed. E-mail: dehaene@shfj.cea.fr

Where does arithmetic come from? For some theorists, the origins of human competence in arithmetic lie in the recursive character of the language faculty (*1*). Chomsky, for instance, stated that "we might think of the human number faculty as essentially an 'abstraction' from human language, preserving the mechanisms of discrete infinity and eliminating the other special features of language" (*2*). Other theorists believe that language is not essential—that humans, like many animals, have a nonverbal "number sense" (*3*), an evolutionarily ancient capacity to process approximate numbers without symbols or language (*4*–*6*) that provides the conceptual foundation of arithmetic. A third class of theories, while acknowledging the existence of nonverbal representations of numbers, postulates that arithmetic competence is deeply transformed once children acquire a system of number symbols (*7*–*9*). Language would play an essential role in linking up the various nonverbal representations to create a concept of large exact number (*10*–*12*).

To elucidate the relations between language and arithmetic, it is necessary to study numerical competence in situations in which the language of numbers is either absent or reduced. In many animal species, as well as in young infants before they acquire number words, behavioral and neurophysiological experiments have revealed the rudiments of arithmetic (*6*, *13*–*16*). Infants and animals appear to represent only the first three numbers exactly. Beyond this range, they can approximate "numerosity," with a fuzziness that increases linearly with the size of the numbers involved (Weber's law). This finding and the results of other neuroimaging and neuropsychological experiments have yielded a tentative reconciliation of the above theories: Exact arithmetic would require language, whereas approximation would not (*12*, *17*–*21*). This conclusion, however, has been challenged by a few case studies of adult brain-lesioned or autistic patients in whom language dysfunction did not abolish exact arithmetic; such a finding suggests that in some rare cases, even complex calculation may be performed without words (*22*).

In the final analysis, the debate cannot be settled by studying people who are raised in a culture teeming with spoken and written symbols for numbers. What is needed is a language deprivation experiment, in which neurologically normal adults would be raised